Venus Transit Parallax Measurement

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Two images of the Venus transit were taken at precisely 22:50:53 UT on June 5, 2012. One was taken by the author from a location in Princeton NJ. The other was taken by Aram Friedman from the top of Haleakala peak on the island of Maui in Hawaii. The two original images are shown in Figure 1.

The two images were loaded into a popular image processing software tool called MaximDL. The smaller image (from Haleakala) was resampled by a factor of 1.58 to make its image scale almost the same as in the NJ image. The images were then "negated" to convert black to white and vice versa. With this inversion, the sunspots look like bright stars against a black background and one can ask MaximDL to align the two images using the "auto star matching" method. The alignment produced this way by MaximDL is excellent. This alignment process is the most error-prone part of the entire analysis. We experimented with different seemingly equivalent methods and got final answers that varied from each other by as much as 10%.

A cropped overlay of the two aligned images is shown in Figure 2.

Using the aligned images, the displacement between the two Venus silhouettes was carefully measured (via a Matlab program) and found to be $(\Delta x, \Delta y) = (19.62, 19.12)$ pixels and so we get

$$\begin{aligned} \text{measured parallax} &= \sqrt{(19.62)^2 + (19.12)^2} \text{ px} \\ &= 27.39 \text{ px} \end{aligned}$$

At the time the images were taken, the Sun's diameter was 31.57 arcminutes. Converting to radians we get:

$$\begin{aligned} 31.57 \ \text{arcmin} &= 31.57 \ \text{arcmin} \times \frac{1 \ \text{degree}}{60 \ \text{arcmin}} \times \frac{\pi \ \text{radians}}{180 \ \text{degrees}} \\ &= 0.009183 \ \text{radians} \end{aligned}$$

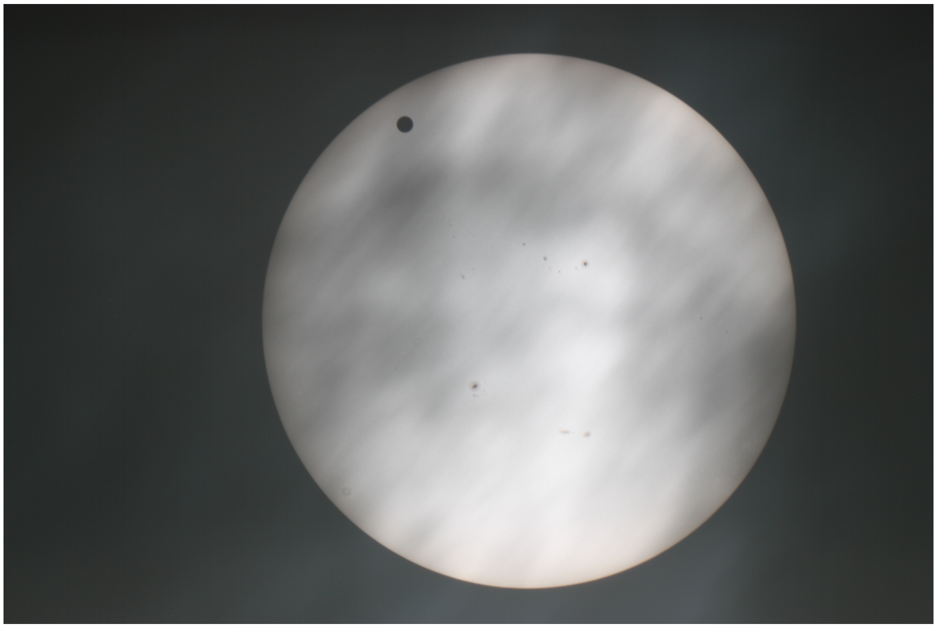

Fig. 1.— *Top:* Image of the Venus transit taken by the author from the top of the North Parking Garage at Princeton University. *Bottom:* Image of the Venus transit taken by Aram Friedman from the top of Mt. Haleakala.

Using the full image, the Sun's diameter was measured to be 2460 pixels. From this, we can convert the measured separation of the two Venus silhouettes to radians:

$$\begin{aligned}\text{measured parallax} &= 27.39 \text{ px} \times \frac{0.009183 \text{ radians}}{2460 \text{ px}} \\ &= 1.0224 \times 10^{-4} \text{ radians} \\ &= 21.09 \text{ arcseconds.}\end{aligned}$$

Using the Sun as a background reference introduces a bias since the Sun itself has a parallax. The true Venus parallax is larger than the Sun-based measurement. To compute how much larger, let x_V denote the distance to Venus and let x_S denote the distance to the Sun. Of course, we are assuming that we don't know these distances in km's but, by Keplers laws, they are known in au's: $x_V = 0.2887$ au and $x_S = 1.0147$ au (as reported by the planetarium program Cartes du Ciel). Similarly, let θ_V and θ_S denote the Venus and Sun true parallaxes, respectively. These quantities are illustrated in Figure 3. From this diagram, we see that

$$\frac{y}{x_V} = \tan(\theta_V) \approx \theta_V, \qquad \text{and} \qquad \frac{y}{x_S} = \tan(\theta_S) \approx \theta_S$$

(using the small-angle approximation for the tangent is fine since the parallax angles are tiny). Solving for y and then eliminating y we get that

$$\theta_V x_V = \theta_S x_S.$$

Hence, we can solve for the Sun's parallax in terms of Venus's parallax:

$$\theta_S = \theta_V \frac{x_V}{x_S}.$$

Let θ denote the *measured parallax* of Venus. It is related to θ_V and θ_S by a simple difference:

$$\theta = \theta_V - \theta_S.$$

Using the equation above expressing θ_S in terms of θ_V, we get

$$\theta = \theta_V \left(1 - \frac{x_V}{x_S}\right).$$

And, finally, we can write the true parallax, θ_V, in terms of the measured parallax, θ:

$$\theta_V = \frac{\theta}{\left(1 - \frac{x_V}{x_S}\right)}.$$

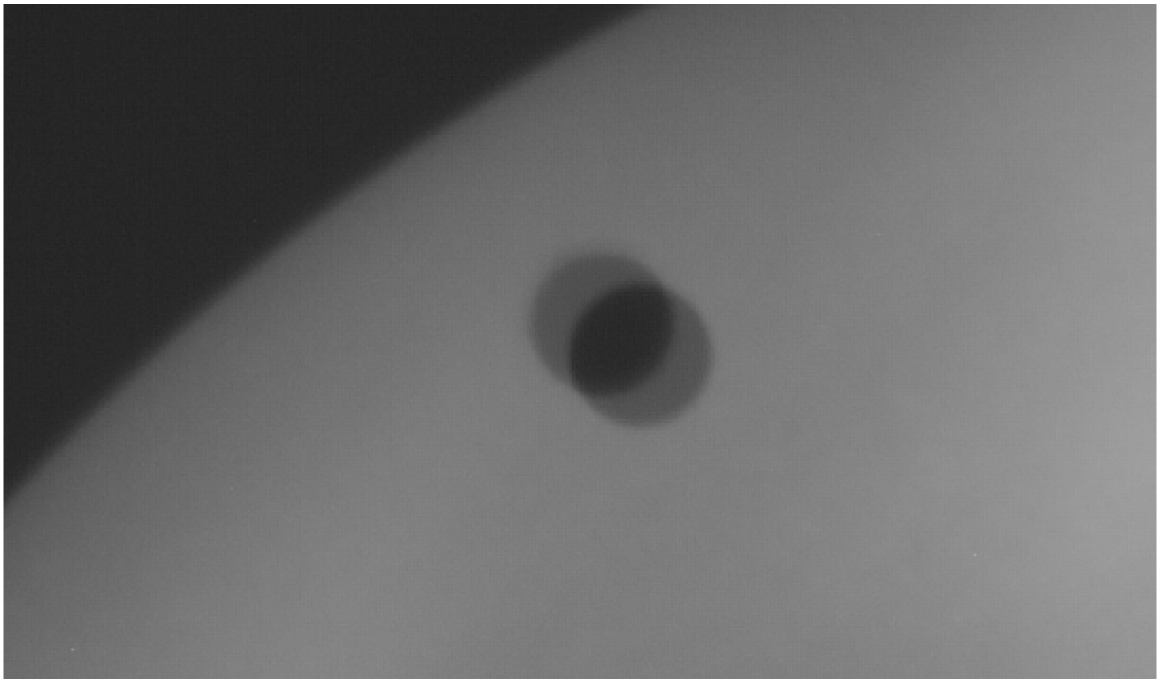

Fig. 2.— The two images resized, aligned, and overlaid.

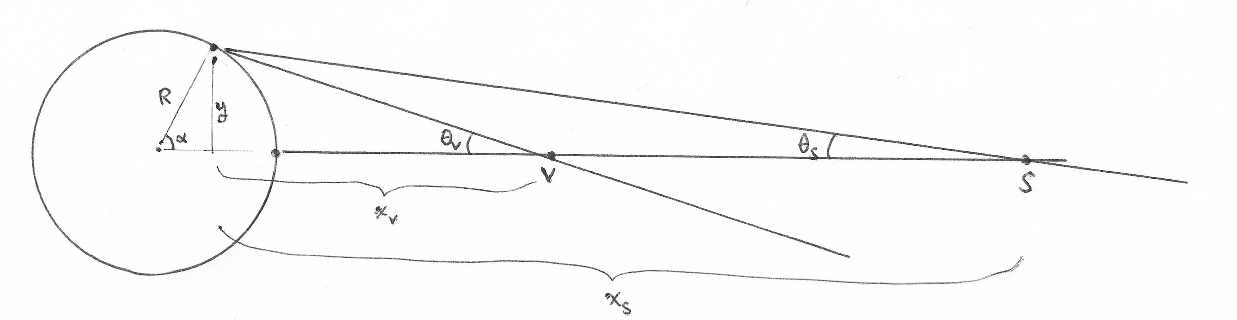

Fig. 3.— The angles θ_S and θ_V are very small and therefore the usual small-angle approximations produce negligible error (for example $\sin(\theta_V) \approx \tan(\theta_V) \approx \theta_V$). The angle α, on the other hand, is not small and therefore the great-circle distance between NJ and HI is not approximately equal to the perpendicular distance y.

Plugging in our numbers, we get that the corrected parallax is

$$\begin{aligned} \text{corrected parallax} &= 1.0224 \times 10^{-4} \text{ radians}/(1 - 0.2887 \text{ au}/1.0147 \text{ au}) \\ &= 1.429 \times 10^{-4} \text{ radians} \end{aligned}$$

Using the planetarium program Celestia, it is easy to see how the Earth looked from the perspective of the Sun at the moment the pictures were taken. (see Figure 4). It is easy then to measure the diameter of the Earth in pixels and also the length of a line segment from Haleakela to Princeton NJ. The former is 742 and the latter is 332. These pixel-scale measurements together with the actual radius of the Earth in kilometers (6378.1 km) allows us to compute the perpendicular distance between Haleakela and NJ in kilometers:

$$\text{Perpendicular distance} = \frac{332}{742/2} 6378.1 \text{ km} = 5707.6 \text{ km}$$

We are finally in a position to compute the distance to Venus. We just need to divide the perpendicular distance by the corrected parallax:

$$\begin{aligned} \text{Distance to Venus} &= 6007 \text{ km}/1.429 \times 10^{-4} \text{ radians} \\ &= 42.04 \text{ million km} \end{aligned}$$

And from this we compute the distance to the Sun:

$$\begin{aligned} 1 \text{ au} &= 42.04 \text{ million km} \times 1 \text{ au}/0.2887 \text{ au} \\ &= 145.6 \text{ million km} \end{aligned}$$

Of course, the astronomical unit is known to be 149.598 million km. Our answer is within about 3% of the correct answer.

Final Remark. Using the 2004 transit event, the European Southern Observatory sponsored a world-wide data gathering campaign and, using the data, got that

$$1 \text{ au} = 149,608,708 \pm 11835 \text{ km}.$$

That project involved 4550 contact timings from 1510 registered observers. The project is summarized here: http://www.eso.org/public/outreach/eduoff/vt-2004/auresults/.

Acknowledgement

I would like to thank Aram Friedman for providing the excellent Hawaii image whose timestamp matched my image to the second. I should also point out that, even though Hawaii is a

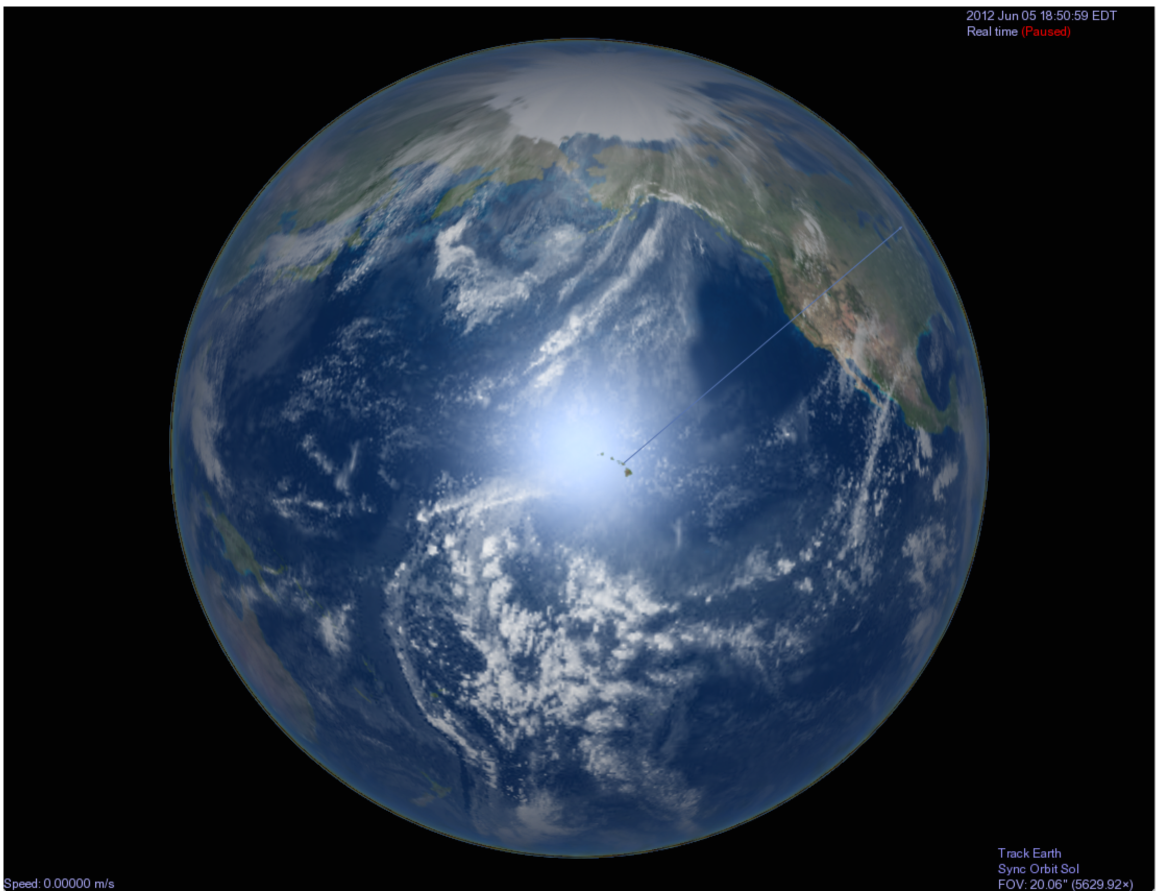

Fig. 4.— Earth as seen from Venus at the time the images were taken (thank you Celestia).

beautiful place to visit, he braved extreme conditions including cold gusty 50 mph (i.e., 80 kph) winds atop the mountain to get his set of excellent images that span the entire transit. I would also like to thank John Church for carefully checking the calculations and pointing out a few improvements to the manuscript.